MAT123

Quadratic Function Model

Quadratics - Intro

The quadratic function, which graphs as a parabola, has two formats.

Format #1: $f(x) = ax^2 + bx + c$

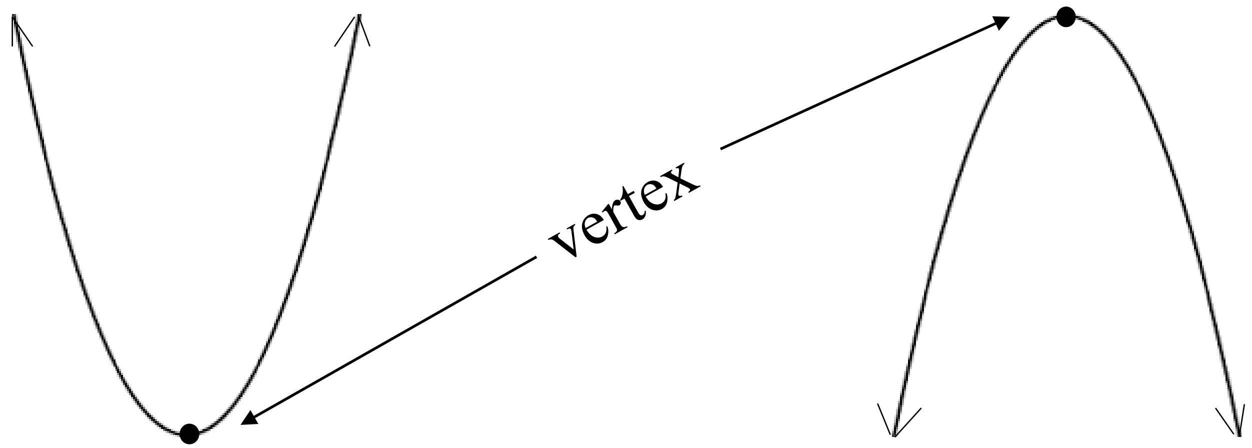
Format #2: $f(x) = a(x-h)^2 + k$

where $a \neq 0$

a called "leading coefficient"

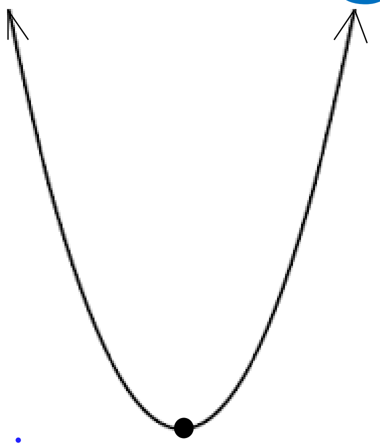
when $a > 0$
parabola opens upward

when $a < 0$
parabola opens downward



Quadratics – Format #1

Format #1: $f(x) = ax^2 + bx + c$ where $a \neq 0$

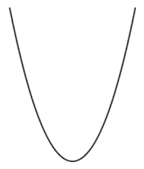


vertex
ordered pair

x $\quad$ y ← evaluate at $x = -\dfrac{b}{2a}$

$$\left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right)$$

expect



$a > 0$

ex. identify components of $f(x) = x^2 + 6x + 10$

vertex x-value: $\quad x = -\dfrac{b}{2a} \to x = -\dfrac{6}{2(1)} = -3$

vertex y-value: $\quad y = f\left(-\dfrac{b}{2a}\right) = f(-3) = (-3)^2 + 6(-3) + 10$

x-value

$$= 9 - 18 + 10$$

$$= -9 + 10 = 1$$

vertex: $(-3, 1)$

y-intercept (there will be exactly 1): $f(0) = 10$ $\quad (0,10)$

set $x = 0$

write intercepts as ordered pairs

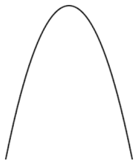
(expect 0-2 x-intercepts) → x-intercept(s): NONE will show why later

set $y = 0$

Quadratics – Format #1 Open Down

Format #1: $f(x) = ax^2 + bx + c$
where $a \neq 0$

$a < 0$

ex. identify components of $h(t) = -t^2 + 2t + 15$ expect 

vertex x-value: $t = -\dfrac{b}{2a} \rightarrow t = -\dfrac{2}{2(-1)} = 1$

vertex y-value: $y = h\left(-\dfrac{b}{2a}\right) = h(1) = -(1)^2 + 2(1) + 15$

$= 16$ Do: simplify

vertex: $(1,16)$

c

y-intercept: $h(0) = 15 \Rightarrow (0,15)$

x-intercept(s): $-t^2 + 2t + 15 = 0$ multiply by $-$

$t^2 - 2t - 15 = 0$ factor

$(t+3)(t-5) = 0$

$t + 3 = 0 \quad t - 5 = 0$

$t = -3 \qquad t = 5 \quad \Rightarrow (-3,0),\ (5,0)$

Quadratic Formula

Sometimes a quadratic does not factor into integers or simple fractions.

Quadratic Formula will ALWAYS work to find x-intercept(s), if any.

Quadratic Formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Now revisit $f(x) = x^2 + 6x + 10.$

$$x = \frac{-6 \pm \sqrt{6^2 - 4(10)}}{2}$$

$$= \frac{-6 \pm \sqrt{36 - 40}}{2}$$

$$= \frac{-6 \pm \sqrt{-4}}{2}$$

negative radical

$\therefore$ no REAL x-intercepts

Revisit x-intercept on $h(t) = -t^2 + 2t + 15.$

$a = -1$ $b = 2$ $c = 15$

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-2 \pm \sqrt{2^2 - 4(-1)(15)}}{2(-1)}$$

$$= \frac{-2 \pm \sqrt{4 + 4(15)}}{-2}$$

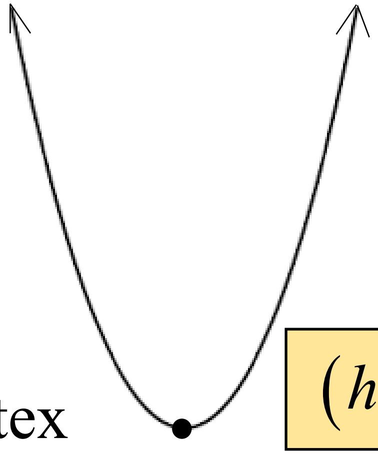
$$= \frac{-2 \pm \sqrt{64}}{-2}$$

$$= \frac{-2 \pm 8}{-2}$$

$$t_1 = \frac{-2 + 8}{-2} = \frac{6}{-2} = -3$$

$$t_2 = \frac{-2 - 8}{-2} = \frac{-10}{-2} = 5$$

Quadratics – Format #2



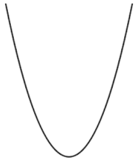
vertex (h,k)

Format #2: $f(x)=a(x-h)^2+k$ where $a \neq 0$

ex. identify components of $f(x)=(x+3)^2+1$

$a=1$

expect



vertex: $(h,k)=(-3,1)$

y-intercept: $f(0)=(0+3)^2+1 \;=(3)^2+1$

$=9+1=10$

$\Rightarrow (0,10)$

x-intercept(s): $(x+3)^2+1=0$

$(x+3)^2=-1$

$\sqrt{(x+3)^2}=\sqrt{-1}$

$\therefore$ no REAL x-intercepts

recall: identify components of $f(x)=x^2+6x+10$

vertex x-value: $x=-\dfrac{b}{2a} \;=-3$

vertex y-value: $y=f\left(-\dfrac{b}{2a}\right) \;=1$

vertex: $(-3,1)$

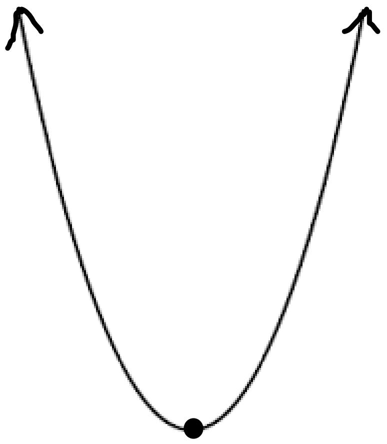
y-intercept: $(0,10)$

x-intercept(s): NONE

Application of the Vertex

Recall the sign of the leading coefficient, a, determines if parabola opens up or down.

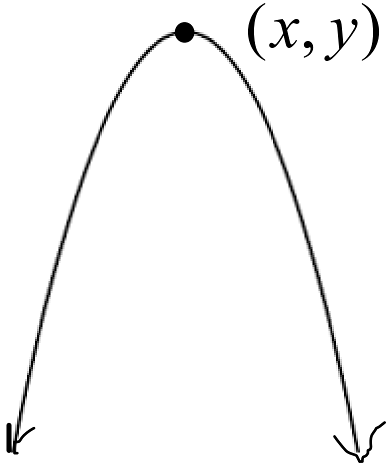
when $a > 0$



vertex is a minimum

when $a < 0$

vertex is a maximum



time

(x, y) or $(t, h(t))$

height

time "x" is <u>when</u> maximum occurs
height "y" is <u>value</u> of maximum

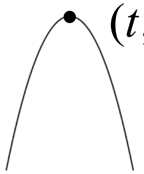
or "maximum value"

Minimum/Maximum of a Quadratic Function

Suppose a ball is tossed straight up in the air from a height of 5 feet with an initial velocity of 20 ft/sec.

This situation is modeled by the function: $h(t) = -16.1t^2 + 20t + 5$

height with respect to time



a. How long until the ball hits the ground? $h(t) = 0 = -16.1t^2 + 20t + 5$ Do: solve for t with quadratic formula

time (t = sec) when $h = 0$

~~$t_1 \approx -.21$ sec~~ $t_2 \approx$ **1.46 sec**

invalid solution: time can't be negative

don't forget units!

exam solution format

(Round Parts a and b to 2 decimal places.)

b. How long until the ball reaches its maximum height? $t = -\frac{b}{2a} = -\frac{20}{2(-16.1)} = \frac{10}{16.1} = \frac{100}{161} \approx$ **.62 sec**

t vertex "x"

.62111180124

DON'T CLEAR CALCULATOR!

c. What is the ball's maximum height? (Round to 1 decimal place.)

vertex "y"

$$h(.6211180124) = -16.1(.6211180124)^2 + 20(.6211180124) + 5$$
$$\approx \boxed{11.2 \text{ feet}}$$

Interpretation: maximum occurs at .62 seconds
maximum value occurs at 11.2 feet

TI instructions:
put function in Y=
Go to Table
put 2nd Ans in X